Aerodynamic Shape Optimization using Free Form Deformation

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Introduction

Aerodynamic Shape Optimization

- Optimization depends on the choice of shape parametrization.
- The ideal shape parametrization technique should be able to describe simple and complex shapes alike and be able to perform robust shape changes using a minimal number of design variables.

Free Form Deformation (FFD)

- A popular technique used extensively in computer graphics and animation.
- The main idea involves enclosing an object in a lattice, and then moving lattice points in order to deform the embedded object.

Mathematical Background

Bezier Curves

- A parametric curve that is central to Free form deformation.
- Given a set of $n+1$ control points, $\vec{P}_i$, a Bezier curve is defined as:

$$\vec{Q}(t) = \sum_{i=0}^{n} B_i^n(t)\vec{P}_i \qquad t\,\epsilon[0,1]$$

where $B_i^n(t)$ is a blending function known as a Bernstein polynomial and is given by

$$B_i^n(t) = \binom{n}{i}(1-t)^{n-i}t^i$$

Bezier Volume

- By taking a tensor product of the Bezier curve, we obtain a Bezier volume:

$$\vec{Q}(s,t,u) = \sum_{j=0}^{n}\sum_{i=0}^{m}\sum_{k=0}^{l} B_i^m(s)B_j^n(t)B_k^l(u)\vec{P}_{ijk}$$

- $\vec{P_{ijk}}$ is the point in the ith row, jth column and kth level of the 3D collection of control points.

Mathematical Background (Continued)

Properties of Bezier Curves

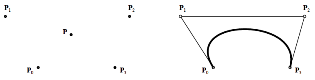
- A Bezier curve with n+1 control points is of degree n.
- Endpoint Interpolation:

$$Q(0) = P_0$$

$$Q(1) = P_n$$

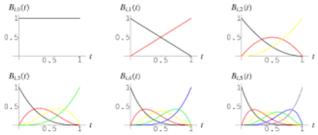
- A Bezier curve is enclosed within the convex hull of its control polygon.
- A displacement of a lattice point results in a global deformation of the curve.
- The n^{th} derivative of the Bezier curve at its start/end depends only the first/last n+1 control points.

A cubic Bezier Curve



(a) Center of mass of four points. (b) Cubic Bézier Curve.

Bernstein Polynomials



Free Form Deformation Algorithm (3D Case)

1. A dimension for the lattice is first specified (ex: 3 x 4 x 5).
2. The lattice points are placed around the object of interest. These lattice points are then considered to be control points of a Bezier volume.
3. Compute the (s,t,u) coordinates of each solid body point in the Bezier volume by solving the following using the Newton-Raphson method:

$$\vec{X} - \sum_{j=0}^{n}\sum_{i=0}^{m}\sum_{k=0}^{l} B_i^m(s)B_j^n(t)B_k^l(u)\vec{P_{ijk}} = 0$$

4. Displace the lattice points. Use the (s,t,u) coordinate of each solid body point in the deformation region to compute its new absolute location:

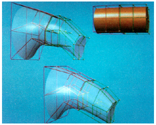
$$\vec{X}_{new} = \sum_{j=0}^{n}\sum_{i=0}^{m}\sum_{k=0}^{l} B_i^m(s)B_j^n(t)B_k^l(u)\vec{P_{ijk}}$$

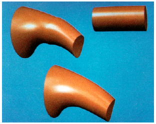
Free Form Deformation (Continued)

Properties of Free Form Deformation

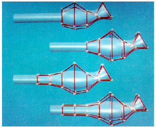
1. The movement of lattice points results in a global deformation of the embedded object.
2. For a local deformation of the object, multiple FFD boxes can be placed together. Geometric continuity between boxes can be imposed using the following rules:
   - For G^0 continuity, two adjoining FFD boxes must have identical control points along the common face.
   - For G^1 continuity, the two planes of control points on either side of the volume must be collinear with points on the common face.

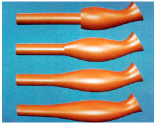
Multiple FFD boxes (for more local control)





Continuity of FFD volumes





Results

Objective

- To test the versatility of Free Form Deformation as an aerodynamic shape optimization tool, the CRM wing-fuselage case was used.
- Two FFD boxes were placed; one around the undercarriage and one around the wing.

Test Cases

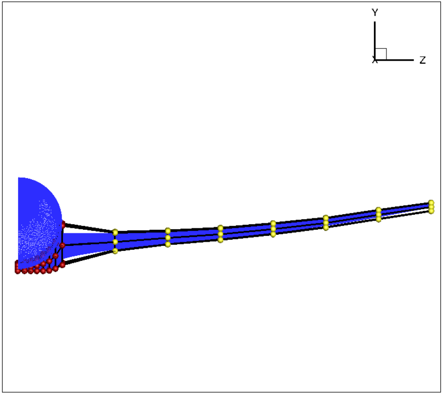
1. The length of the undercarriage was decreased.
2. The angle of the wing was changed.
3. The width of the undercarriage was decreased.
4. An exaggerated increase in the volume of the undercarriage was done.

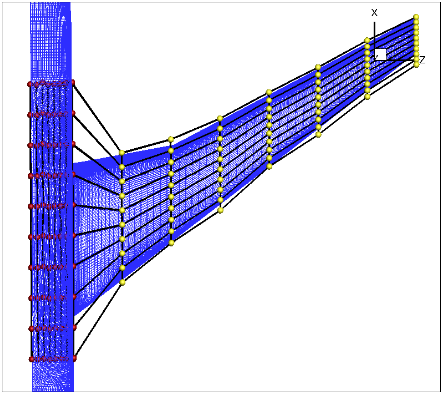
Notes

- In images in which two solid bodies are shown, the blue one shows the deformed state and the red depicts the undeformed.
- A set notation will be used to denote which lattice points were moved to cause the deformation. ex: If the i={2} set of points were moved, all points on row 2 of the lattice were moved.

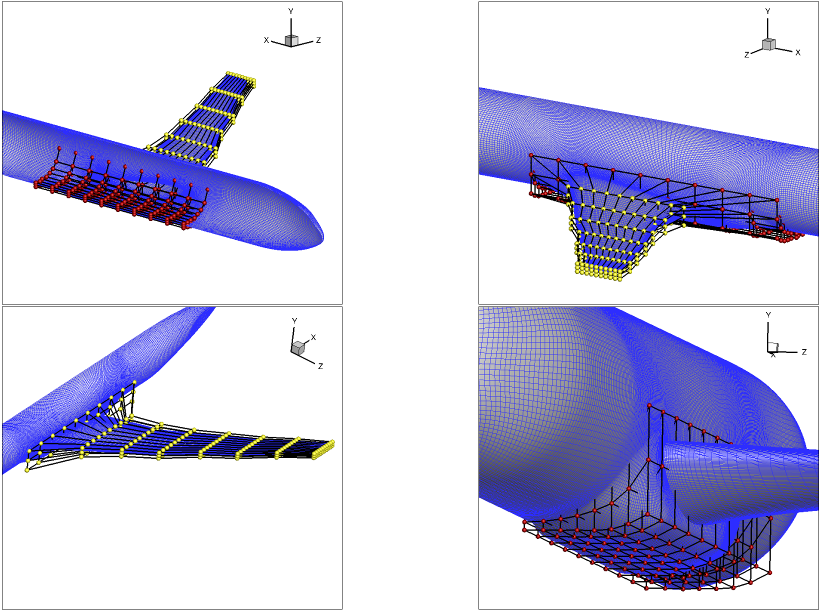
Initial FFD Lattice

The lattice around the undercarriage and wing are both 10 x 3 x 8 (10 points along the x direction, 3 along the y direction and 8 along the z direction)



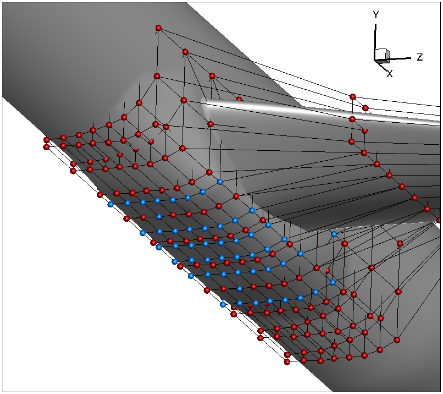


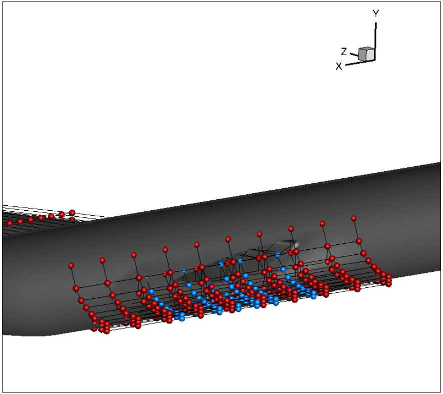
Initial FFD Lattice (Continued)



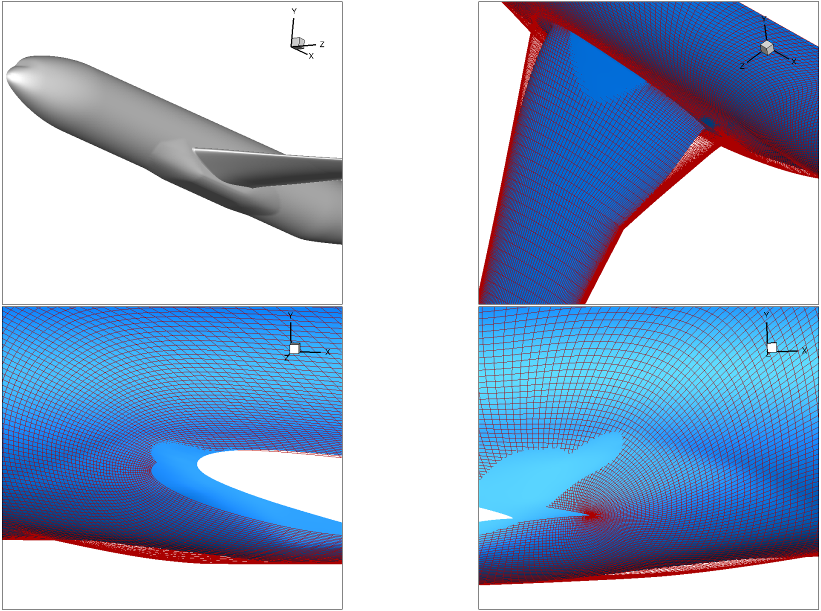
Undercarriage Length Decrease Deformation

- All FFD points except for the i = {1,10} j = {3} set of points for the undercarriage were displaced. The distance in the x direction of all points to the center of the undercarriage was decreased by 40 percent.



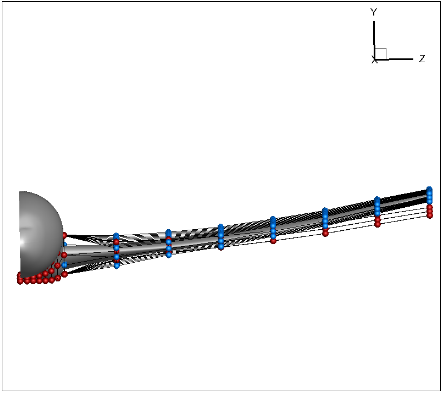


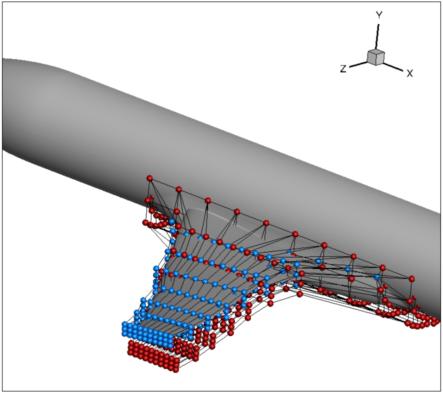
Undercarriage Length Decrease Deformation (Continued)



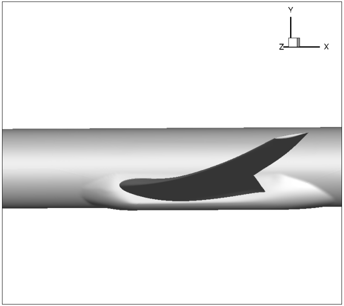
Wing Angle Twist Deformation

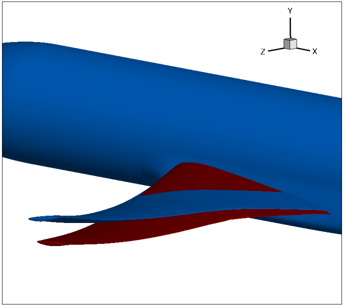
- All FFD points of the wing lattice, except for the k = {1} set of points, were rotated about the center axis of the wing by 5 degrees.
- The i = {2 - 10 }, j = {2} and k = {8} set of points for the undercarriage FFD lattice were rotated about the center axis of the wing by 5 degrees.

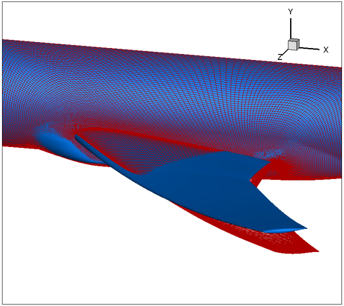


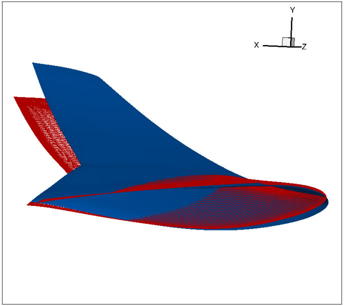


Wing Angle Twist Deformation (Continued)



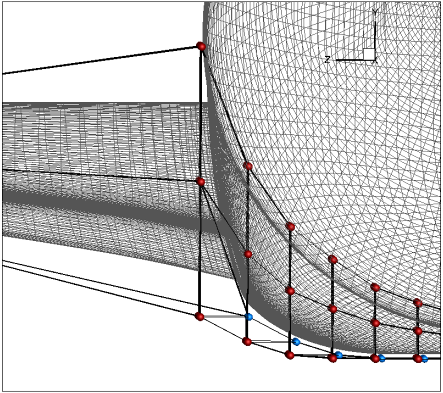


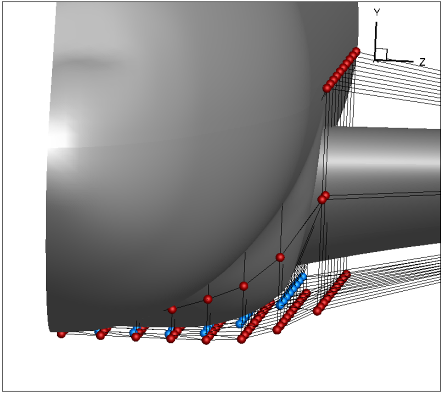




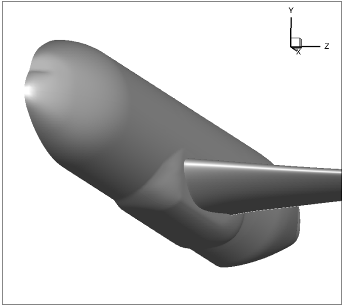
Undercarriage Width Decrease Deformation

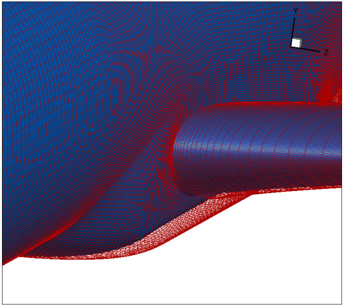
- All FFD points except for the i = {1,10} j = {2,3} and k = {1} set of points for the undercarriage were displaced in the negative z direction by 20 units.

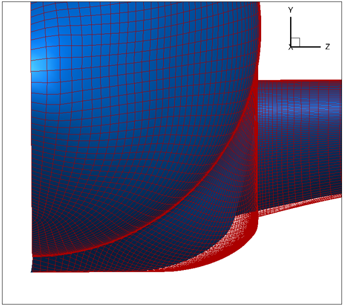


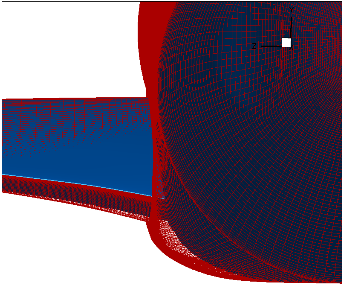


Undercarriage Width Decrease Deformation (Continued)

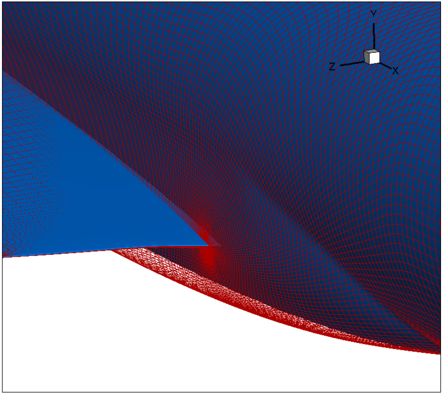


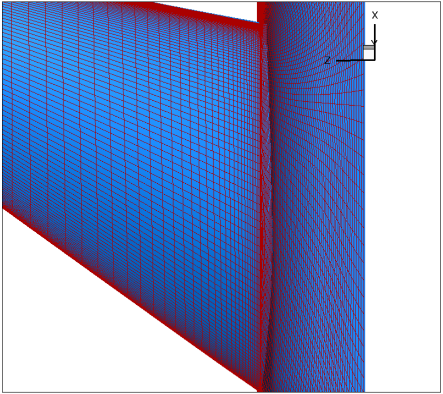






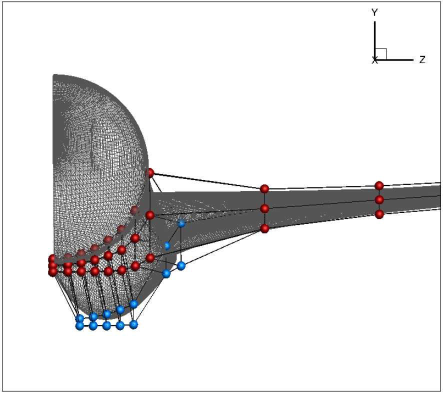
Undercarriage Width Decrease Deformation (Continued)

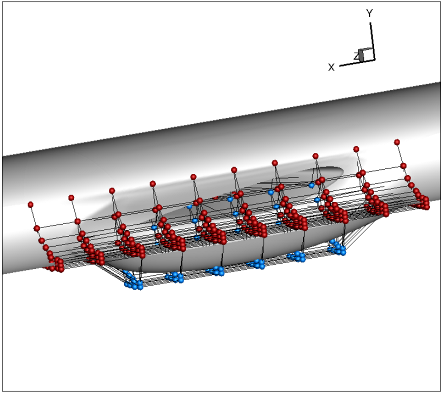




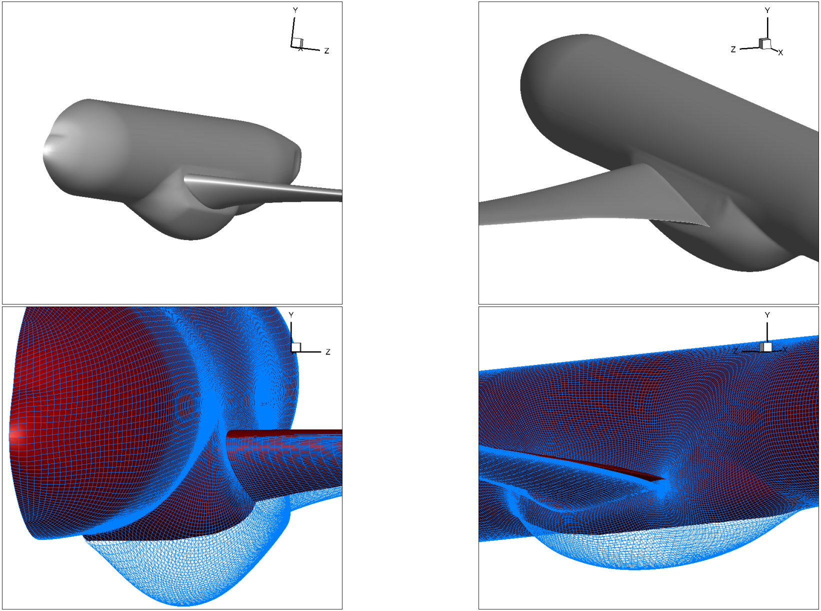
Exaggerated Undercarriage Volume Increase Deformation

- All FFD points except for the i = {1,2,9,10}, j = {3} and k = {1,7,8} set of points for the undercarriage were displaced down by 70 units and in the positive z direction by 15 units. The k = {7,8} set of points were displaced down by 10 units and in the positive z direction by 40 units.

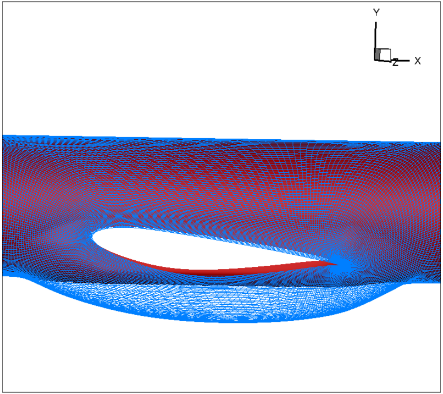


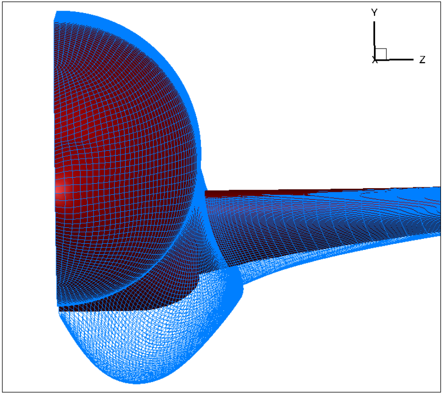


Exaggerated Undercarriage Volume Increase Deformation (Continued)



Exaggerated Undercarriage Volume Increase Deformation (Continued)





Conclusion

- Free Form Deformation is a versatile method that can be used in the field of aerodynamic shape optimization.
- The key procedure in Free Form Deformation is to surround an object with a collection of lattice points, and then displace these points in order to deform the shape of the object.
- These lattice points can then be used as design variables during optimization.
- Multiple FFD boxes can be placed on objects in which separate regions will be optimized differently.

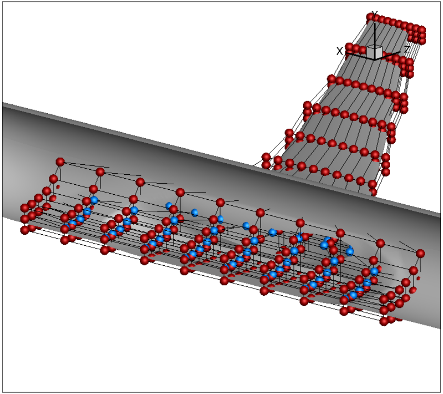
Extra Cases

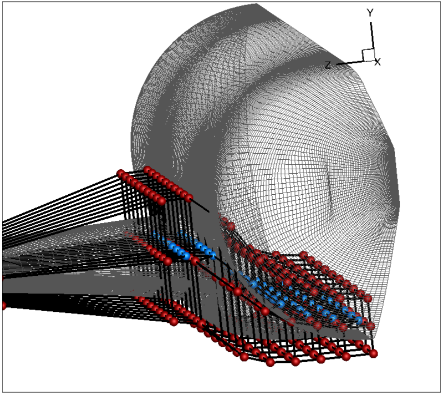
Extra Test Cases

1. The width at the midsection of the undercarriage was decreased.
2. The dihedral angle of the wing was changed.

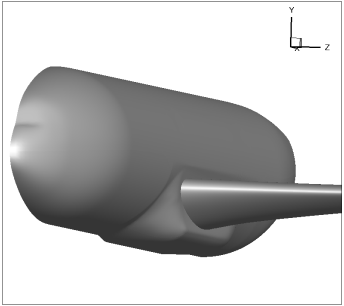
Mid Undercarriage Width Decrease Deformation

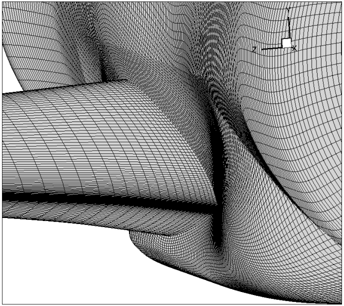
- All FFD points except for the j = {1,3} and k = {1} set of points for the undercarriage were displaced in the negative z direction by 20 units.

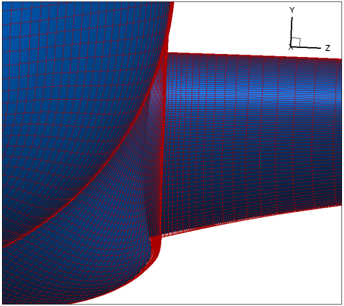


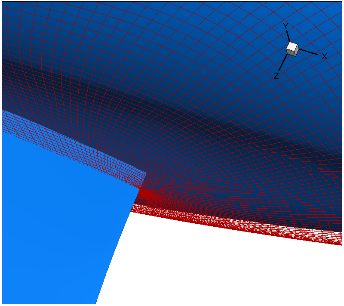


Mid Undercarriage Width Decrease Deformation (Continued)



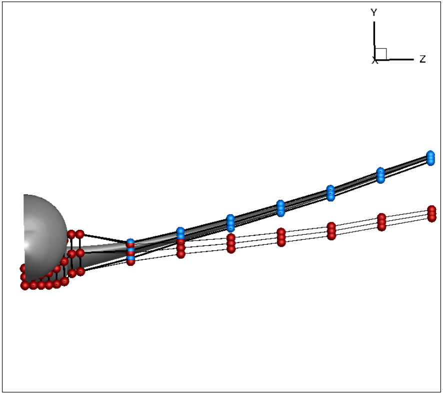


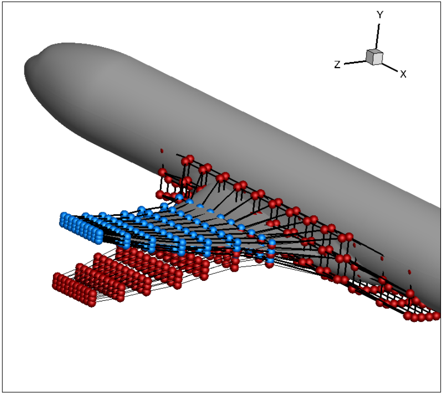




Wing Dihedral Deformation

- All FFD points except for the k = {1} set of points for the wing were displaced in the positive z direction. The dihedral angle was increased by 10 degrees.





Wing Dihedral Deformation (Continued)